# Phase Noise

There are also **short-term** instabilities (e.g., msec to $\mu$sec) in oscillator frequency!

We can model these as:

$$v_c(t) = a\cos\left[\omega_0 t + \phi_n(t)\right]$$

where the relative phase $\phi_n(t)$ is a random process called **phase noise**.

**Q:** *It looks a lot like **phase modulation**!*

**A:** Essentially, it **is**.

The **random** process $\phi_n(t)$ has a small magnitude, *i.e.*:

$$\left|\phi_n(t)\right| \ll 1$$

Note since the phase changes as a function of time, the **frequency** will as well! Specifically:

$$\begin{aligned}\omega(t) &= \frac{d\left(\omega_0 t + \phi_n(t)\right)}{dt} \\ &= \omega_0 + \frac{d\phi_n(t)}{dt} \\ &= \omega_0 + \omega_n(t)\end{aligned}$$

where:

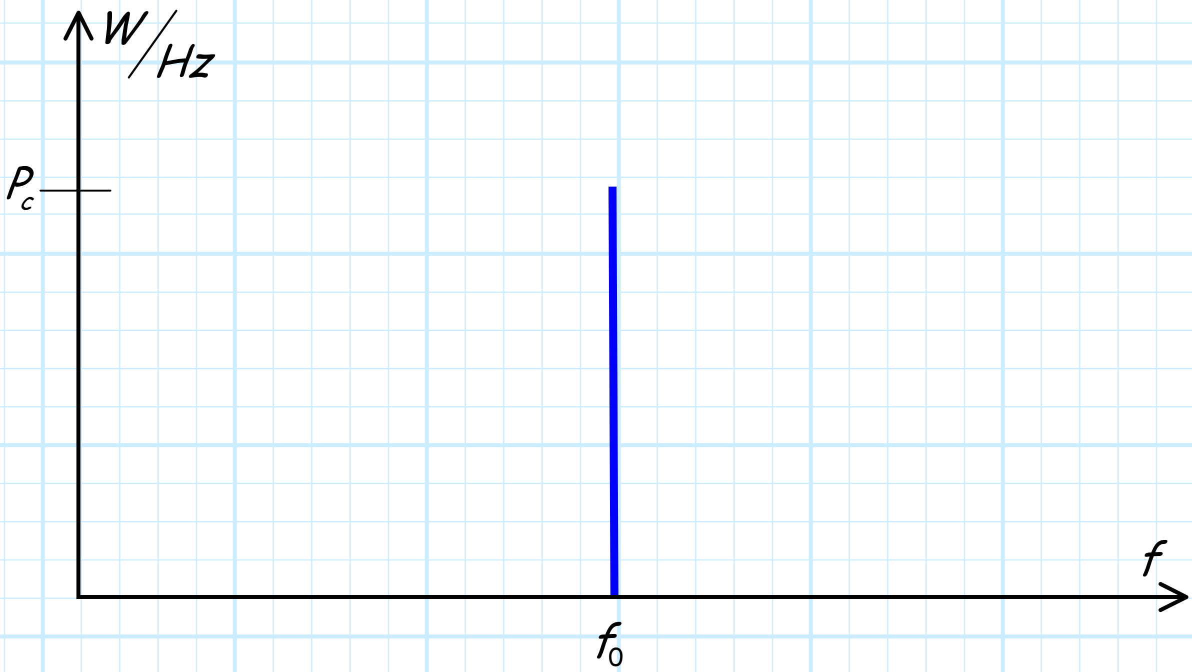
$$\omega_n(t) = \frac{d\phi_n(t)}{dt}$$

As a result, the **frequency** of the oscillator is also a **random** process.

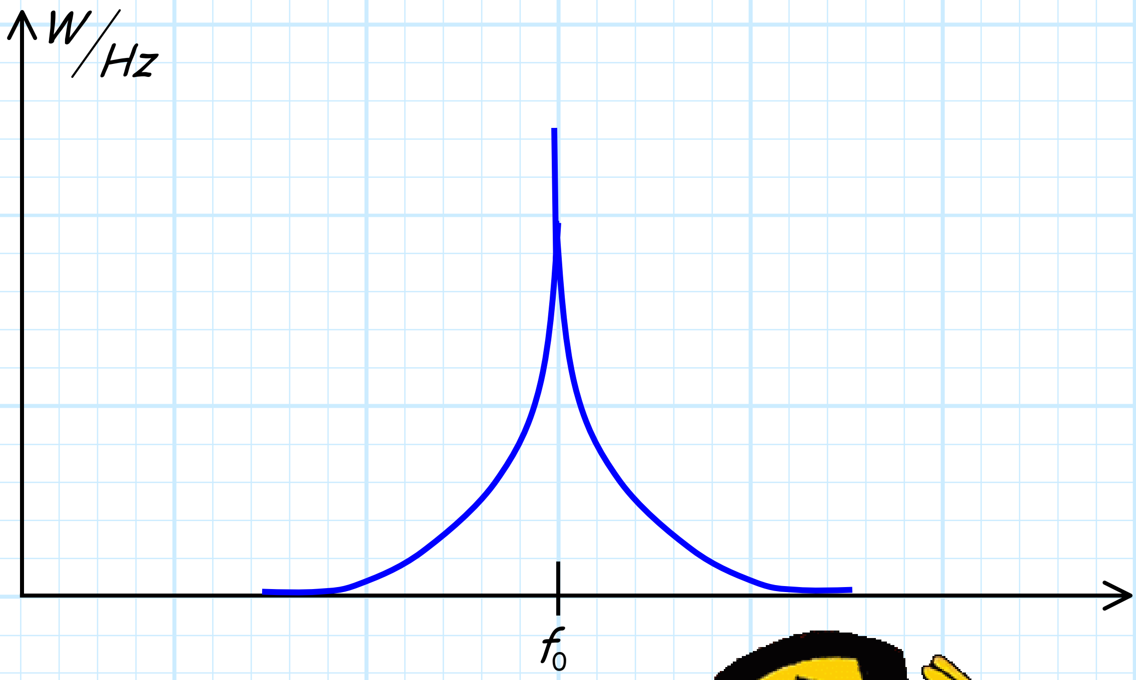
*I.E., the oscillator frequency changes* ***randomly*** *as a function of time!*

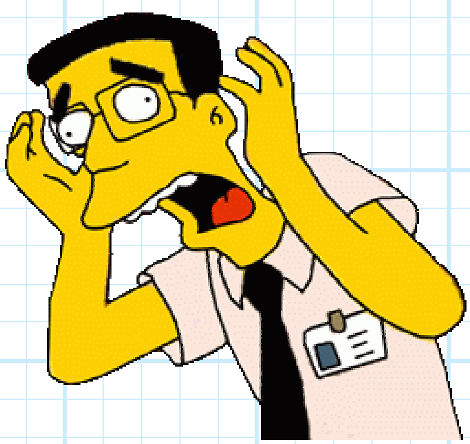
This random fluctuation **spreads** the oscillator signal **spectrum**.

In other words, **instead** of the spectrum of a **perfect**, "pure" tone:



we get a wider, **imperfect** spectrum:



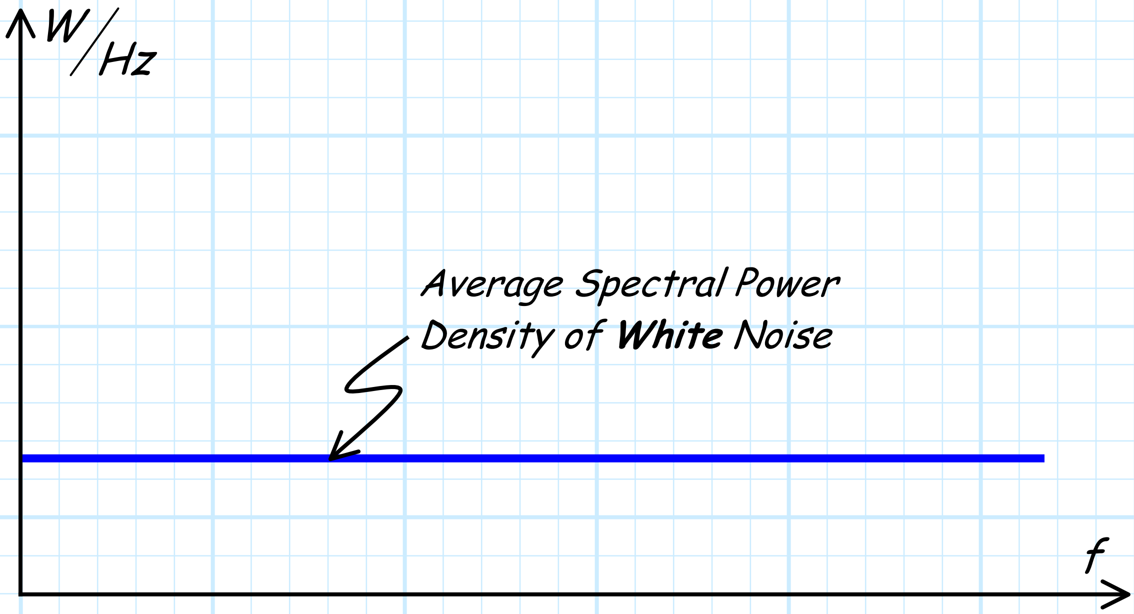



In this case, we say our oscillator has **spectral impurities**!

* Since the phenomenon of phase noise is a random process, we must describe the signal spectrum in terms of its **average** spectral power density.

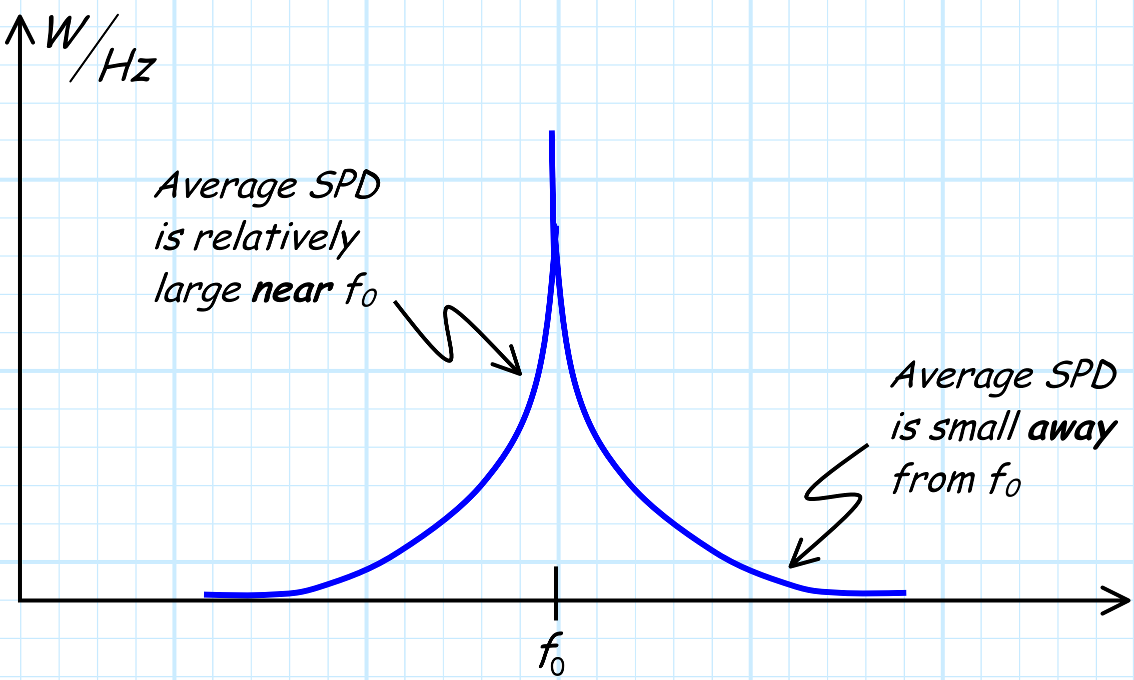
* Spectral Power Density is expressed in units of **Watts/Hz**.

* For **white** noise, the spectral power density is a **constant** with respect to frequency:




* However, for **phase** noise, the resulting spectral power density **changes** as a **function** of frequency!

Specifically, the average spectral power density of an oscillator **increases** as frequency $f$ **nears** the nominal signal (i.e., **carrier**) frequency $f_0$.



Now, although we typically express average spectral power density in Watts/Hz or dBm/Hz, we generally express the spectral power density of an **oscillator** output in **dBc** !

In other words, we are only concerned about the magnitude of the phase noise spectral power density **in comparison to the oscillator signal power** $P_c$ !

* Note we have a mathematical **problem** here! $P_c$ is in **Watts**, and SPD is in **Watts/Hz**. Therefore, the ratio of the two is **not** unitless!

* We get around this problem by specifying the noise as its power in a **1 Hz bandwidth**.

→ **Numerically**, this is identical to the average spectral power density of the noise!

For **example**, if the noise power has an average spectral power density 2.0 $\mu W/Hz$, then the noise power in a bandwidth of $\mathbf{1}\,Hz$ is:

$$2.0\frac{\mu W}{Hz}\left(1\ Hz\right)=2.0\ \mu W$$

Thus, phase noise is expressed as a rather **cumbersome**:

$dBc$ in a 1 Hz bandwidth

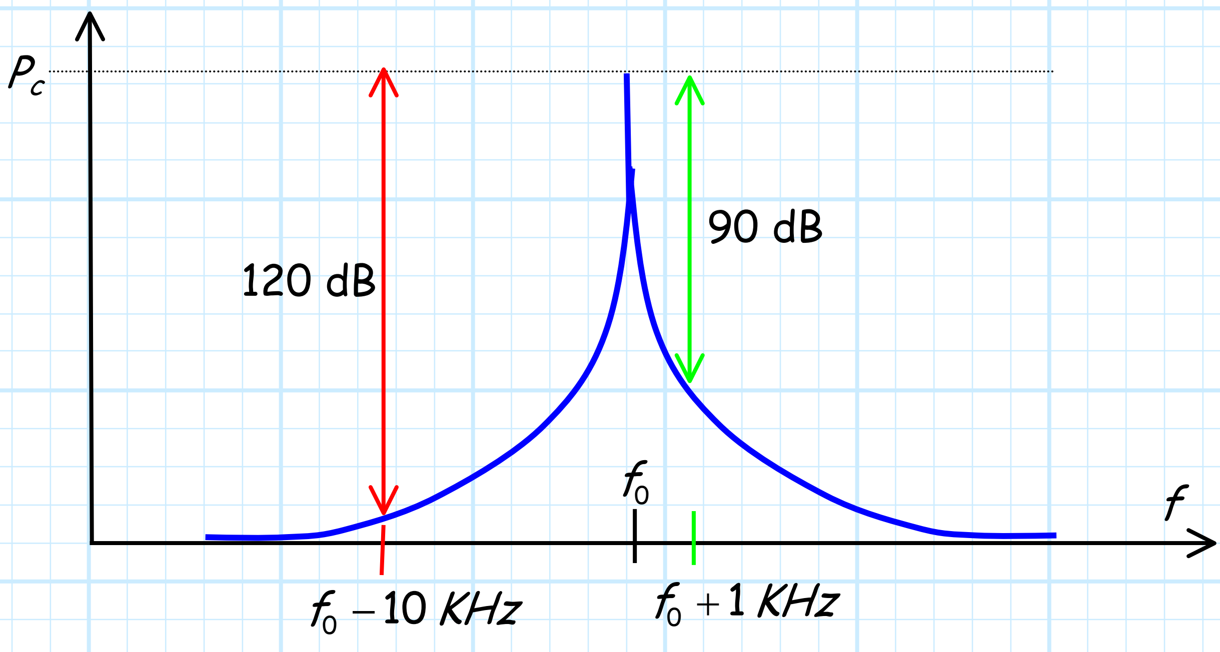
**Q:** *But phase noise is a **function** of frequency $f$. Do we have to **explicitly** specify this function?*

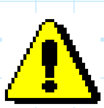
**A:** Generally speaking **no**. Phase noise is generally specified by stating the value of the noise power at **one** or **two** specific frequencies, with **respect** to the carrier frequency $f_0$.

Typically, the frequencies where the phase noise is **specified** ranges from 1 KHz to 100 KHz from the carrier.

For example, a **typical** oscillator spec might say:

*-90 dBc in a 1 Hz bandwith at 1 KHz from the carrier, **and***
*-120 dBc in a 1 Hz bandwith at 10 KHz from the carrier.*



Make sure that **you** know how to proper specify the phase noise of an oscillator. It is **often** incorrectly done, and the source of many **lost points** on an exam or project!